Quantum theory of Cherenkov radiation in laser plasmas

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Introduction

As an especial dielectric medium, plasma has seductive potential to generate rich electromagnetic radiation and as a consequence shall become an ideal work-medium for producing high-power radiations with various wavelengths due to its unrestrained damage threshold. In current development, Cherenkov radiation generated from plasmas is of attention-getting application in the beamed high-power emission and diagnosis of plasma parameters due to its aptotic distribution of emitting angle (Cherenkov geometry).

Constitutionally, the radiation is a quantum problem from its microcosmic mechanism. To understand the physical essence of radiation process in depth, imaginably, the development of a fully quantum mechanical description is necessary. Just so, the quantization of the electromagnetic field in dielectrics has received considerable attention in last decade. However the current theories are still limited to isotropic non-absorbing media, which cannot hold true to the anisotropic absorbing media especially to the plasma created by the high-intensity ultrashort pulse laser. Our aim is to extend the current theory into the anisotropic absorbing media by utilizing the Langevin noise (LN) approach. In our method we let Langevin force described by the fluctuation dissipation theorem is Langevin noise and introduce the dielectric function that satisfy the universal Kramers-Kronig relation into the classical phenomenological Maxwell equations, and then transfer all the quantities of the classical electromagnetic field into relevant quantum mechanical operators. Finally the quantization of radiation field in the isotropic absorbing medium was realized.

As an application, we employed the present quantum theory to describe the Cherenkov radiation stimulated by an uniformly moving charge particle in the anisotropic medium and obtained the Cherenkov radiation intensity.

Field quantization in medium

We start from field operators which can be regarded as obeying Maxwell's equations in the forms

$$\nabla\times\hat{\boldsymbol{E}}(\boldsymbol{r},t)=-\frac{\partial\hat{\boldsymbol{B}}(\boldsymbol{r},t)}{\partial t}$$

$$\nabla\times\hat{\boldsymbol{B}}(\boldsymbol{r},t)=\mu_0\frac{\partial\hat{\boldsymbol{D}}(\boldsymbol{r},t)}{\partial t}+\mu_0\hat{\boldsymbol{J}}(\boldsymbol{r},t)$$

Where the electric- and magnetic-field operators are related to the vector potential operator by

$$\hat{\boldsymbol{E}}(\boldsymbol{r},t)=-\frac{\partial\hat{\boldsymbol{A}}(\boldsymbol{r},t)}{\partial t}$$

$$\hat{\boldsymbol{B}}(\boldsymbol{r},t)=\nabla\times\hat{\boldsymbol{A}}(\boldsymbol{r},t)$$

Then, the vector potential operator satisfies

$$\sum_\beta\left[k^2c^2-\varepsilon(\omega)\omega^2\right]\delta_{\alpha\beta}\hat{A}_\beta(\boldsymbol{k},\omega)=\mu_0c^2\hat{J}_\alpha(\boldsymbol{k},\omega)$$

Using Green function method, it allows us to express $\hat{A}(\boldsymbol{k},\omega)$ In terms of $\hat{J}(\boldsymbol{k},\omega)$ as

$$\hat{A}_\alpha(\boldsymbol{k},\omega)=\frac{1}{\varepsilon_0}\sum_\beta G^{\Lambda}_{\alpha\beta}(\boldsymbol{k},\omega)\hat{J}_\beta(\boldsymbol{k},\omega)$$

Taking advantage of forms of Green function in isotopic medium and anisotropic medium , quantized expression of vector potential operator can be obtained

$$\hat{A}^{\Lambda}_\alpha(\boldsymbol{r},t)=$$

$$-\frac{\mathrm{i}}{4\pi\varepsilon_0^2}\int_0^{+\infty}\mathrm{d}\omega\int\mathrm{d}^3\boldsymbol{k}\left\{\sum_\beta G^{\Lambda}_{\alpha\beta}(\boldsymbol{k},\omega)\hat{J}^{+}_\beta(\boldsymbol{k},\omega)\exp[-\mathrm{i}(\omega t-\boldsymbol{k}\cdot\boldsymbol{r})]-\mathrm{H.c.}\right\}$$

Quantum theory of Cherenkov in medium

Consider a charge particle of mass m and electric Charge e unifornly moving in a medium., Hamiltonian of the combined system is

$$\mathrm{H}=\mathrm{H}_{rad}+\mathrm{H}_{ele}+\mathrm{H}_{\mathrm{I}}$$

The explicit firm of H_{I} for creation of a single-photon process is of the form

$$\mathrm{H}_{\mathrm{I}}=-\frac{e}{m}P\cdot\hat{A}$$

The interaction Hamiltonian induces transitions between these states. The transition amplitude in the first-order approximation is

$$M_{if}=\langle 1_k|\langle q-k|\mathrm{H}_{\mathrm{I}}|q\rangle|0\rangle$$

The transition probability per unit time for creation of a photon is given by Fermil's golden rule

$$\Gamma_{i\to f}=\frac{2\pi}{\hbar}\left|M_{if}\right|^2\delta\left(E_f-E_i\right)$$

Results and discussion

Cherenkov radiation intensity in isotopic medium

$$\frac{dw_x^{\Lambda}}{dt}=$$

$$\frac{e^2v_\perp^2}{4\pi^3\varepsilon_0}\int_0^{+\infty}d\omega\int d^3k\frac{\omega^3\,\mathrm{Im}\,\varepsilon(\omega)}{\left[k^2c^2-\omega^2\varepsilon(\omega)\right]^2}\delta\left[k\cdot v-\omega\left(1+\frac{\hbar^2k^2}{2m}\right)\right]$$

Cherenkov radiation intensity in anisotropic medium

$$\frac{dw_x^{\Lambda}}{dt}=$$

$$\frac{e^2v^2}{4\pi^3\varepsilon_0}\int_0^{+\infty}d\omega\omega\int d^3k\left\{\frac{\omega^2\,\mathrm{Im}\,\varepsilon_{11}(\omega)}{\left[k^2c^2-\omega^2\varepsilon_{11}(\omega)\right]^2}-\frac{\omega^6\,\mathrm{Im}\,\varepsilon_{11}(\omega)\left[|\varepsilon_{12}(\omega)|^2+|\varepsilon_{13}(\omega)|^2\right]}{\left[k^2c^2-\omega^2\varepsilon_{11}(\omega)\right]^4}\right.$$

$$+\frac{\omega^6\left[\varepsilon_{12}(\omega)\varepsilon^*_{21}(\omega)\right]\left[k^2c^2-\omega^2\varepsilon_{33}(\omega)\right]^2\mathrm{Im}\,\varepsilon_{22}(\omega)}{\left[k^2c^2-\omega^2\varepsilon_{11}(\omega)\right]\left[k^2c^2-\omega^2\varepsilon_{22}(\omega)\right]\left[k^2c^2-\omega^2\varepsilon_{33}(\omega)\right]^2}\left(1-\frac{\omega^4|\varepsilon_{21}(\omega)|^2+\omega^4|\varepsilon_{23}(\omega)|^2}{\left[k^2c^2-\omega^2\varepsilon_{22}(\omega)\right]^2}\right)$$

$$\left.+\frac{\omega^6\left[\varepsilon_{13}(\omega)\varepsilon^*_{31}(\omega)\right]\left[k^2c^2-\omega^2\varepsilon_{22}(\omega)\right]^2\mathrm{Im}\,\varepsilon_{33}(\omega)}{\left[k^2c^2-\omega^2\varepsilon_{11}(\omega)\right]\left[k^2c^2-\omega^2\varepsilon_{22}(\omega)\right]\left[k^2c^2-\omega^2\varepsilon_{33}(\omega)\right]}\left(1-\frac{\omega^4|\varepsilon_{31}(\omega)|^2+\omega^4|\varepsilon_{32}(\omega)|^2}{\left[k^2c^2-\omega^2\varepsilon_{33}(\omega)\right]^2}\right)\right\}$$

$$\times\delta\left[k\cdot v-\omega\left(1+\frac{\hbar^2k^2}{2m}\right)\right]$$

The results showed that, the medium dielectric tensor changes the density of the Cherenkov radiation but does not change the emitting angle of the radiation, which is of advantage for the collection of radiation wave with expectant wavelengths and the diagnosis of plasma parameters.

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