



# Analytical theory for the laser driven TNSA ion acceleration

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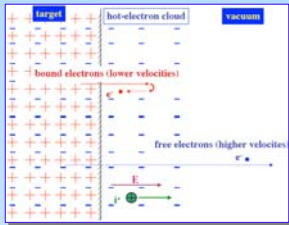
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**Abstract:** A theoretical model of the quasistatic electric field, formed at the rear surface of a thin solid target irradiated by a ultraintense subpicosecond laser pulse, due to the appearance of a cloud of ultrarelativistic bound electrons, is developed. It allows one to correctly describe the spatial profile of the accelerating field and to predict maximum energies and energy spectra of the accelerated ions. The agreement of the theory with the experimental data looks satisfactory in a wide range of conditions. Provisions of regimes achievable in the future are given.

## The kinetic isothermal model of bound (negative energy) electrons



$$f_e(\mathbf{r}, \mathbf{p}) = C \exp\left(-\frac{\varepsilon(\mathbf{r}, \mathbf{p})}{T_e}\right)$$

$$n_{bound}(\mathbf{r}) = \iiint_{\varepsilon(\mathbf{r}, \mathbf{p}) \leq 0} d^3 p f_e(\mathbf{r}, \mathbf{p})$$

$$\varepsilon(\mathbf{r}, \mathbf{p}) \leq 0 \Rightarrow |\mathbf{p}| \leq p_{max}(\mathbf{r}) \quad \varphi = \frac{e\phi}{T_e}, \quad v = \frac{n}{\tilde{n}}$$

$$\nabla^2 \varphi = v_{bound}(\varphi)$$

## The 1-dim ultra-relativistic (UR) case

$$f_e(x, p) = \frac{\tilde{n}}{2mcK_1\left(\frac{mc^2}{T}\right)} \exp\left(-\frac{\varepsilon(x, p)}{T_e}\right) \quad \varepsilon(x, p) = mc^2(\gamma - 1) - e\phi(x) \leq 0$$

$$\Rightarrow \gamma \leq \gamma_{max}(x) \equiv 1 + \frac{e}{mc^2} \phi(x)$$

with  $p^2 \leq p_{max}^2 \equiv m^2 c^2 \left[ \left(\frac{e\phi}{mc^2}\right)^2 + \frac{2e\phi}{mc^2} \right]$

for  $|\mathbf{p}|/mc \gg 1 \quad f_e^{UR}(x, p) = \tilde{n} \frac{c}{2T_e} \exp\left(-\frac{c|\mathbf{p}| - e\phi(x)}{T_e}\right) \quad p^2 \leq p_{max}^2 \equiv \left(\frac{e\phi}{c^2}\right)^2$

## Maximum ion energy

$$\varphi_0 = \frac{(\varphi^* - 1)e^{\varphi^*} + 1}{(e^{\varphi^*} - 1)}$$

$K_{i,max} = Z\varphi_0 T_e$  maximum ion energy  
 $\varepsilon_{e,max} = \varphi^*$  normalized maximum electron energy

## Energy spectrum of ions accelerated in the UR hot-electron cloud

- conservation of the n. of ions in phase space  $n_i(\varepsilon_i) d\varepsilon_i = n_i(\xi) d\xi$
- from a thin ion layer initially placed between  $\xi = 0$  and  $\xi = \Delta\xi$

$$n_i(\varepsilon_i) = \frac{H(\varepsilon_i - Z\varphi_0) - H(\varepsilon_i - Z\varphi_0 - Z\Delta\varphi)}{\sqrt{2Z} \left[ \exp\left(\frac{\varepsilon_i}{Z}\right) - \frac{\varepsilon_i}{Z} - 1 \right]^{1/2}}$$

## The maximum electron energy $\varepsilon_{e,max} = \varphi^*$ as a scaling law

- from the analysis of several published results we get the fitting

$$\varepsilon_{e,max} = \frac{K_{e,max}}{T_e} = A + B \ln[E_L(J)]$$

$$A = 4.8, B = 0.8$$

where  $E_L$  is the laser energy  
 [M. Passoni and M. Lontano, *Phys. Rev. Lett.* **101**, 115001 (2008)]

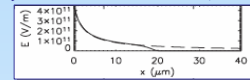
## Results

### Spatial behavior far from the boundary

for  $\xi \gg 1 \quad |\varphi(\xi)| \equiv \left| \frac{e\phi(x)}{T_e} \right| \ll 1 \quad \varphi(\xi) \approx \varphi_0 \left( 1 - \frac{\xi}{\xi_f} \right)^4$  where  $\xi_f = \sqrt{6\pi^{1/2}} \varphi_0^{1/4}$

the potential vanishes, with its low-order derivatives, electric field, density, at a finite  $\xi = \xi_f$

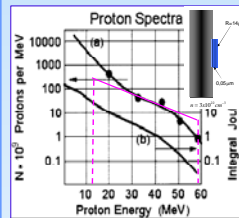
[L. Romagnani, et al., *Phys. Rev. Lett.* **95**, 195001 (2005)]



[M. Lontano and M. Passoni, *Phys. Plasmas* **13**, 042102 (2006)]

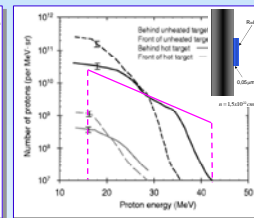
### Simulations of ion energy spectra

$$D = 2R = f_L + w_i \tan \theta$$



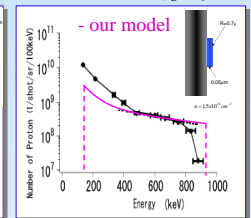
[R.A. Snavely, et al., *Phys. Rev. Lett.*, **85**, 2945 (2000)]

$$E_L = 500 \text{ J} \quad w_i = 100 \mu\text{m} \quad I_L = 3 \times 10^{20} \text{ W/cm}^2 \quad \tau_L = 0.5 \text{ ps}$$



[P. McKenna, et al., *Phys. Rev. E*, **70**, 036405 (2004)]

$$E_L = 400 \text{ J} \quad w_i = 100 \mu\text{m} \quad I_L = 2 \times 10^{20} \text{ W/cm}^2 \quad \tau_L = 0.7 \text{ ps}$$

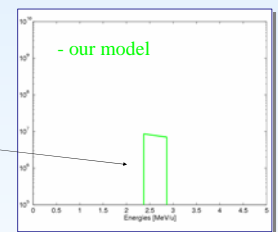
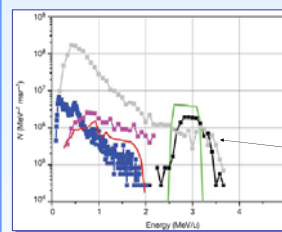


[M. Nishiuchi, et al., *Phys. Lett. A*, **357**, 339 (2006)]

$$E_L = 0.25 \text{ J} \quad w_i = 3 \mu\text{m} \quad I_L = 3 \times 10^{18} \text{ W/cm}^2 \quad \tau_L = 70 \text{ fs}$$

### Quasi-monoenergetic MeV carbon beams

[B. M. Hegelich et al., *Nature* **439**, 441 (2006)]

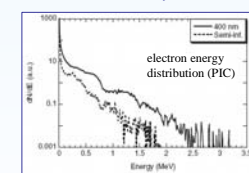
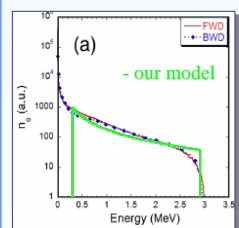


$$E_L = 20 \text{ J} \quad I_L \approx 10^{19} \text{ W/cm}^2 \quad w_i = 20 \mu\text{m} \quad \tau_L = 0.8 \text{ ps}$$

**C<sup>5+</sup> ions**  
 estimated layer thickness: < 5 nm

### Use of Ultrahigh-Contrast Laser Pulses

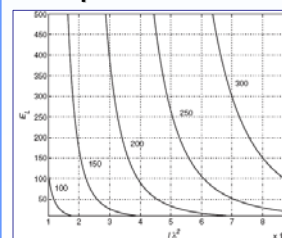
[T. Ceccotti et al., *Phys. Rev. Lett.* **99**, 185002 (2007)]



$$E_L = 0.65 \text{ J} \quad w_i = 0.4 \mu\text{m} \quad I_L = 5 \times 10^{18} \text{ W/cm}^2 \quad \tau_L = 0.065 \text{ ps} \quad \text{contrast} > 10^{10}$$

No fitting parameters used for these experiments!

### Expected laser intensity/energy values for hadron-therapy



A possibility for 250 MeV protons:

- Ti:Sa system ( $\lambda = 0.8 \mu\text{m}$ )
- $E_L = 50 \text{ J}$ ;  $\tau_L = 5 \text{ fs}$ ;  $f_L = 7 \mu\text{m}$
- $I_L = 1 \times 10^{22} \text{ W/cm}^2$ ; 10 PW system

